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Optimal production strategy of bimetallic deposits under technical and economic uncertainties using stochastic chance-constrained programming

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Abstract
In order to catch up with reality, all the macro-decisions related to long-term mining production planning must be made simultaneously and under uncertain conditions of determinant parameters. By taking advantage of the chance-constrained programming, this paper presents a stochastic model to create an optimal strategy for producing bimetallic deposit open-pit mines under certain and uncertain conditions. The uncertainties of grade, price per product, and capacities of the various stages in the process of production of the final product were considered. The results of solving the deterministic and stochastic models showed that the stochastic model had a greater compatibility and performance than the other ones.

Keywords: *Bimetallic Deposits, Uncertainty, Grade, Price, Capacity.*

1. Introduction
The existence of a good strategy in a production process sometimes has such benefits that in the case of its absence, the production organizations active in the field of industries and mines might deviate from the direction of healthy growth and survival in a competitive environment. Often the goal of a production plan and strategy in an open-pit mine is to achieve the maximum net present value of cash flows. In this regard, production planning is one of the crucial steps in the process of planning an open-pit mine [1, 2]. Therefore, the main tasks for mine planning are collecting different quality and quantity data and formulating the best possible strategy and planning [3].
Devising a planning strategy for the production of two products is among the most important problems faced by mine designers in bimetallic deposits. Instead of the conventional production plans, new production strategies compatible with the terms of such deposits are highly important. In addition to the diversity and number of its extant metals, problems in bimetallic deposits such as uncertainty in grade are also significant. The necessity and importance of a production strategy determine the production planning purposes to some extent. This strategy can be followed in wide time frames from short-term to long-term. Among the existing criteria and goals, maximizing the economic value, providing a feed with specified tonnage and potential grade, delaying stripping in the early years of mining, and minimizing deviations in the mine planning purposes are highly significant [4]. Given that the optimal production strategies are considered among modern problems dependent on production planning, and as little research work has been done in this regard, this work addresses important dependent problems, i.e. production planning and cut-off grade that are closely related.
Two types of mathematical methods are used to solve the problem of production planning: deterministic and uncertainty-based methods. In deterministic models, it is assumed that all inputs have a fixed known value, although assumptions are not always realistic. Methods based upon uncertainty have been used since 1990 in order to optimize the design and open-pit mine production

planning. In these methods, the true value of some data such as ore grade, product prices, and costs of production can vary within a certain range [5]. The uncertainty methods include linear programming methods based on uncertainties and integer random number planning.

In relation to production planning, the research works conducted by Gershon [5], Dagdelen and Johnson [6], and Caccetta and Hill [7] have provided models for mine production planning based upon deterministic planning. These models are less consistent than the uncertainty models, which consider actual conditions. The works of Abdollahisharif et al. [8], Ataei and Osanloo [9], Asad and Topal [10], Gholamnejad [11], and Bascetin and Nieto [12] have investigated cut-off grade, which is closely related to the production strategy. All decisions associated with production planning and cut-off grade determination should be adopted by considering the most effective parameters under uncertainty conditions to catch up with reality. However, uncertainties have not been taken into account in any of them. In the work of Dimitrakopoulos and Ramazan [13], production planning has been optimized under uncertainty. For information on similar works in the areas of production planning and cut-off grade in uncertainty, the paper of Dimitrakopoulos and Goodfellow [14] can be cited.

In this research work, the optimal production strategy is conducted under very important uncertain parameters such as price, grade, and the capacity of various parts of the production process because, in reality, there is no possibility of conditions occurring precisely in a certain form, and the production process always faces deterministic conditions. Thus it is necessary to investigate the positive and negative effects of these conditions on planning. This research work indicates that the uncertainties in geological conditions, production, and market should be considered to have an optimal production strategy in mines. Correspondingly, this work provides a stochastic model to optimize the production process of bimetallic deposits using chance constrained programming planning. The most obvious differences between this work and the others include the attention given to the production process of bimetallic deposits and the simultaneous effects of stochastic parameters of grade, product price, and the capacities of mine, concentrator (processing) plant, and refinery.

2. Optimal model of bimetallic production strategy

2.1. Expressing theoretical concepts of model

The process of producing metals from extracted ores was considered according to Figure 1 in order to present an optimal planning model for a bimetallic deposits production strategy. With the aim of maximizing the net present value, the suggested model determines the metals produced, the materials sent through different stages from the mine to the processing plant, and finally, to the refinery plant. In other words, the purpose of the model is to optimize the process of producing the final product while increasing the profitability in sales and decreasing the executive costs.

The suggested model is based upon two assumptions of certainty and uncertainty in the key parameters of the problem. Uncertainty caused by the variable conditions of the market and the executive process of producing the product is considered so that the model is consistent with the real world conditions. First, according to indicators, parameters, and decision variables, the deterministic model is expressed. Then to match the conditions of the problem with reality, by considering the uncertainties in grade, selling price, and capacity of each stage using chance constrained programming [15], the deterministic model is developed on the basis of stochastic programming. Accordingly, the assumption is the determination of stochastic parameter distribution as normally distributed.

Such a model in the literature about stochastic programming is called “chance-constrained programming”. If different confidence levels are defined for the constraints, it would be called “singular chance-constrained programming”, and in case a similar confidence level is considered for a group of constraints, it is called “joint chance-constraint programming” [16].

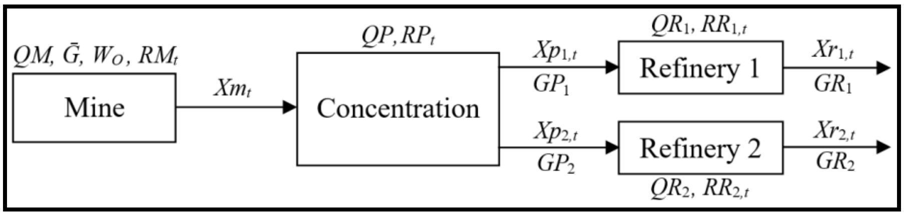

Figure 1. Process and main stages of producing final product from mineral ore.

2.2. Deterministic model

While formulating the problem, joint consideration is given to the indices, parameters, and decision variables for the deterministic and uncertain parts of the problem, according to Table 1.

In order to model the problem process according to Figure 1, first, the relationships among the stages of this process are expressed as the existent communication in the field of the rate of transferred material and on the basis of the recovery rate and cut-off grade, as in Equations 1-3.

$$Xm_t = W_O \times RM_t \times \bar{G} \qquad \forall t \tag{1}$$

$$Xp_{i,t} = Xm_t \times RP_t \times \left(\bar{G} \big/ GP_i \right) \qquad \forall i,t \tag{2}$$

$$Xr_{i,t} = Xp_{i,t} \times RR_{i,t} \times \left(GP_i \big/ GR_i \right) \qquad \forall i,t \tag{3}$$

Equation 1 determines the rate of material transferred from the mine to the processing plant based on the extraction recovery stage and average in-situ grade. Equation 2 expresses the rate of concentrate sent from the processing plant to the refinery sector based on the recovery of this and the previous stages. Finally, Equation 3 calculates the rate of pure metal i'th based on the two previous stages and materials received. According to the equations above and the concept of NPV, an objective function is expressed for the maximization of the current net value according to Equation 4 and by taking into account constraints of the problem. This equation represents maximization of profits or the difference of income from selling pure metals and all different stages and fixed costs during the investigation period. It should be noted that the objective function is expressed with attention to the three relationships mentioned according to the original variable of the problem, i.e. the rate of a pure metal or the final product i'th in the period t'th. In other words, if it is necessary to calculate Xm_t and $Xp_{i,t}$, only the rate of pure metal i'th in the period t'th($Xr_{i,t}$) should be obtained and inserted in Equations 1-3.

Table 1. Indices, parameters, and decision variables for mathematical modeling.

Index	i: Index for products (metals); i = 1,2,…,M t: Index for periods over production strategy; t = 1,2,…,T
Parameter	RM_t: Mine stage recovery over period t RP_t: Processing stage recovery over period t $RR_{i,t}$: Refinery stage recovery of metal i over period t $\bar{G}$: Average grade GR_i: Grade of metal i in refinery stage GP_i: Grade of metal i in processing stage W_O: Ore deposit tonnage QM: Annual capacity of mine (tonnes) QP: Annual capacity of processing (tonnes) QR_i: Annual capacity of refinery for metal i (tonnes) CM_t: Annual mining cost over period t CP_t: Annual processing cost over period t $CR_{i,t}$: Annual refining cost of metal i over period t F_t: Annual fixed costs over period t d_t: Annual discount rate over period t $P_{i,t}$: Price of metal i over period t $U_{i,t}$: Upper bound of price for metal i over period t $L_{i,t}$: Lower bound of price for metal i over period t
Decision variable	Xm_t: Amount of ore that can be sent from mine to processing plant over period t $Xp_{i,t}$: Amount of concentration that can be sent from processing plant to refinery over period t $Xr_{i,t}$: Amount of metal (final product) i that can be refined over period t

$$Max\,(NPV) = \sum_{t=1}^{T}\sum_{i=1}^{M}\left[\frac{\left(P_{i,t} - CR_{i,t}\right)\cdot Xr_{i,t} - CP_t \cdot Xp_{i,t} - CM_t \cdot Xm_t - F_t}{\left(1+d_t\right)^t}\right]$$

$$= \sum_{t=1}^{T}\sum_{i=1}^{M}\left[\frac{\left[\left[Xr_{i,t}\left(\left[(P_{i,t} - CR_{i,t})(RP_t.RR_{i,t}.GP_i.\bar{G})\right] - (CP_t \cdot \bar{G}.RP_t.GR_i) - (CM_t \cdot GP_i.GR_i)\right)\right](RP_t.RR_{i,t}.GP_i.\bar{G})^{-1}\right] - F_t}{\left(1+d_t\right)^t}\right] \tag{4}$$

s.t.

$$Xm_t \leq QM \quad \forall t \tag{5}$$

$$\sum_{i=1}^{M} Xp_{i,t} \leq QP \quad \forall t \tag{6}$$

$$Xr_{i,t} \leq QR_i \quad \forall i,t \tag{7}$$

$$Xp_{i,t} \leq Xm_t \times RP_t \times \left(\bar{G}/GP_i\right) \quad \forall i,t \tag{8}$$

$$Xr_{i,t} \leq Xp_{i,t} \times RR_{i,t} \times \left(GP_i/GR_i\right) \quad \forall i,t \tag{9}$$

$$Xm_t, Xp_{i,t}, Xr_{i,t} \geq 0 \quad \forall i,t \tag{10}$$

In the deterministic model provided, Equation 5 expresses the constraints of the annual capacity of the mine. This limitation ensures that the amount of ore extracted each year of the mine's life cannot exceed the maximum capacity of the mine in any given year. In Equation 6, the constraint of the maximum rate of concentrate produced from a total bimetallic in the processing sector that must be sent to the refinement sector is formulated. This constraint ensures that the total annual produced bimetallic concentrate in the processing sector cannot exceed the maximum annual capacity of the processing plant. The constraint mentioned in Equation 7 ensures that the annual production of each of the final products (pure metals) does not exceed the annual capacity of the refined sector. Equations 8 and 9 are considered based upon Equations 2 and 3 in order to maintain the relationships among the capacity of different stages of the production process, as shown in Figure 1, in the model. Finally, the constraints of the lowest value of decision variables or, in other words, the minimum rate of any of the products of the mine sectors, processing, and refinery, are written in Equation 10.

2.3. Stochastic model

In order to achieve results close to reality, the uncertainties in grade parameters of ore, price, and capacity of the mine sectors of processing and refinement of the problem in this work are considered based upon the stochastic chance-constrained programming approach and according to Table 2.

According to the process of chance-constrained programming and using the parameters in Table 2, the objective function and constraints of the stochastic model are formulated using Equations 11-20.

Table 2. Random parameters for stochastic mathematical modeling.

Parameter	Description		
$\bar{G}$	$\mu_{\bar{G}}$: Average grade	$\sigma^2_{\bar{G}}$: Grade variance	θ : Confidence level for grade
QM	μ_{QM} : Average mine capacity	σ^2_{QM} : Variance of mine capacity	γ : Confidence level for mine production capacity
QP	μ_{QP} : Average processing capacity	σ^2_{QP} : Variance of processing capacity	β : Confidence level for processing capacity
QR_i	μ_{QR_i} : Average refinery capacity	$\sigma^2_{QR_i}$: Variance of refinery capacity	α_i : Confidence level for refinery capacity
$P_{i,t}$	$\mu_{P_{i,t}}$: Average price for each metal	$\sigma^2_{P_{i,t}}$: Variance of price for each metal	$\lambda_{i,t}$: Confidence level for price for each metal

$$Max\ E(NPV) = \\ = \sum_{t=1}^{T}\sum_{i=1}^{M}\left[\frac{\left[\left[Xr_{i,t}\left(\left[(\mu_{P_{i,t}} - CR_{i,t})(RP_t.RR_{i,t}.GP_i.\bar{G})\right] - (CP_t \cdot \bar{G}.RP_t.GR_i) - (CM_t \cdot GP_i.GR_i)\right)\right](RP_t.RR_{i,t}.GP_i.\bar{G})^{-1}\right] - F_t}{(1+d_t)^t}\right] \quad (11)$$

$$Xm_t \le \mu_{QM} + (Z_\gamma \times \sigma_{QM}) \quad \forall t \quad (12)$$

$$\sum_{i=1}^{M} Xp_{i,t} \le \mu_{QP} + (Z_\beta \times \sigma_{QP}) \quad \forall t \quad (13)$$

$$Xr_{i,t} \le \mu_{QR_i} + (Z_{\alpha_i} \times \sigma_{QR_i}) \quad \forall \mathrm{i,t} \quad (14)$$

$$\frac{GP_i \times Xp_{i,t}}{Xm_t \times RP_t} \le \mu_{\bar{G}} + (Z_\theta \times \sigma_{\bar{G}}) \quad \forall i,t \quad (15)$$

$$Xr_{i,t} = Xp_{i,t} \times RR_{i,t} \times \left(GP_i / GR_i\right) \quad \forall i,t \quad (16)$$

$$\mu_{P_{i,t}} + (Z_{\lambda_{i,t}} \times \sigma_{P_{i,t}}) \le U_{i,t} \quad \forall i,t \quad (17)$$

$$\mu_{P_{i,t}} + (Z_{\lambda_{i,t}} \times \sigma_{P_{i,t}}) \ge L_{i,t} \quad \forall i,t \quad (18)$$

$$Xm_t, Xp_{i,t}, Xr_{i,t} \ge 0 \quad \forall i,t \quad (19)$$

Equations 12-14, respectively, represent the random constraints equal to the deterministic equations 5-7 based upon uncertainties in the mine's capacity, processing, and refinery. Equation 15 is used to express the deterministic constraint of Equation 8 randomly. Due to the lack of random parameters in the constraint of Equation 9 in the deterministic model, the same equation is repeated with no changes in Equation 16 of the stochastic model. Constraints 17 and 18 indicate the randomness of metal selling prices and that this parameter should not exceed the acceptable and permitted limit of the market. Equation 19 is the replicate of Equation 10 in the deterministic model.

3. Results and discussion

3.1. Studied data

In order to investigate the results and performance of the presented models, the data of a case example was used as in Table 3. The data is related to an assumed bimetallic deposit that is considered similar to the actual data in a bimetallic deposit. Parameters related to the capacity of the mine, processing, and refinery, their recovery rates, discount rates, and fixed costs per year for 18 years in a row are shown in this table.

Table 4 shows the values for the parameters of grade, capacity, and statistical coefficients related to the refinery sector for the two desired metals. Also the parameter values related to the grades, capacities, and statistical coefficients related to the mine and processing sectors have been brought for the assumed ore deposit in Table 5.

Table 4. Values related to different parameters and coefficients of refinery sector for two metals.

Parameter	*i* = 1	*i* = 2
GR_i	0.99	0.92
GP_i	0.65	0.80
QR_i	3800	1400
μ_{QR_i}	3800	1400
σ_{QR_i}	417.80	277.87
α_i	0.95	0.95

Table 5. Values related to parameters and coefficients of the mine and processing sectors.

Parameter	Value
$\bar{G}$	0.54
$\mu_{\bar{G}}$	0.54
$\sigma_{\bar{G}}$	0.21
θ	0.95
QM	8000000
μ_{QM}	8000000
σ_{QM}	1144183.4
γ	0.95
QP	6800000
μ_{QP}	6800000
σ_{QP}	919096.5
β	0.95
W_O	140000000

Table 3. Data related to an assumed mine for case study analysis.

Parameter \ t		1	2	3	…	6	7	8	…	11	12	13	…	16	17	18
RM_t		0.950	0.970	0.960	…	0.960	0.950	0.960	…	0.950	0.950	0.960	…	0.960	0.970	0.960
RP_t		0.615	0.620	0.635	…	0.650	0.625	0.630	…	0.695	0.600	0.665	…	0.690	0.675	0.680
CP_t		5.32	5.30	5.31	…	5.42	5.45	5.41	…	5.63	5.64	5.67	…	5.68	5.70	5.74
CM_t		2.50	2.55	2.53	…	2.56	2.63	2.61	…	2.62	2.64	2.64	…	2.73	2.67	2.77
F_t		980000	980000	980000	…	980000	980000	980000	…	980000	980000	980000	…	980000	980000	980000
d_t		6.9	7.1	7.1	…	7.9	7.9	8.1	…	8.4	8.3	8.1	…	8.4	8.5	7.9
$RR_{i,t}$	$i = 1$	0.955	0.965	0.925	…	0.960	0.935	0.970	…	0.980	0.980	0.975	…	0.980	0.985	0.985
	$i = 2$	0.555	0.565	0.525	…	0.560	0.535	0.570	…	0.580	0.580	0.575	…	0.580	0.585	0.585
$CR_{i,t}$	$i = 1$	93	94	94	…	93	97	99	…	96	96	96	…	95	96	95
	$i = 2$	276	279	277	…	280	277	272	…	275	276	279	…	278	279	277
$P_{i,t}$	$i = 1$	1918	1911	1928	…	1961	1959	1953	…	1979	1971	1967	…	1951	1966	1975
	$i = 2$	9116	9136	9122	…	9211	9232	9219	…	9207	9191	9241	…	9240	9221	9233
$U_{i,t}$	$i = 1$	2598	2584	2521	…	2588	2562	2567	…	2599	2530	2577	…	2574	2576	2550
	$i = 2$	12230	12244	12248	…	12804	12847	12839	…	12826	12830	12890	…	12900	12881	12887
$L_{i,t}$	$i = 1$	1610	1605	1618	…	1677	1669	1659	…	1686	1677	1675	…	1666	1674	1681
	$i = 2$	7208	7212	7209	…	7255	7289	7288	…	7285	7279	7302	…	7318	7303	7299
$\mu_{P_{i,t}}$	$i = 1$	1918	1911	1928	…	1961	1959	1953	…	1979	1971	1967	…	1951	1966	1975
	$i = 2$	9116	9136	9122	…	9211	9232	9219	…	9207	9191	9241	…	9240	9221	9233
$\sigma_{P_{i,t}}$	$i = 1$	308	306	310	…	284	290	294	…	293	294	292	…	285	292	294
	$i = 2$	1114	1108	1126	…	1593	1615	1620	…	1619	1639	1649	…	1660	1660	1654
$\lambda_{i,t}$	$i = 1$	0.95	0.95	0.95	…	0.95	0.95	0.95	…	0.95	0.95	0.95	…	0.95	0.95	0.95
	$i = 2$	0.95	0.95	0.95	…	0.95	0.95	0.95	…	0.95	0.95	0.95	…	0.95	0.95	0.95

3.2. Analysis of results

After entering the deterministic and stochastic models presented into the Lingo 14.0 software and placing the assumed deposit data shown in Tables 3-5, both models were resolved and the output results of the software processing were presented in Table 6.

Table 6. Software output results obtained from processing models.

	Deterministic model					Stochastic model				
Decision variable	Xm_t	$Xp_{i,t}$		$Xr_{i,t}$		Xm_t	$Xp_{i,t}$		$Xr_{i,t}$	
t		*i*=1	*i*=2	*i*=1	*i*=2		*i*=1	*i*=2	*i*=1	*i*=2
1	8000000	3899099	2900901	2444.814	1400	9882181	4463877	3848037	2798.941	1857.096
2	8000000	3950442	2849558	2502.944	1400	9882181	4531984	3779930	2871.401	1857.096
3	8000000	3733333	3066667	2267.340	1400	9882181	4243989	4067925	2577.473	1857.096
4	8000000	3975439	2824561	2531.832	1400	9882181	4565141	3746773	2907.395	1857.096
5	8000000	4000000	2800000	2560.606	1400	9882181	4597721	3714192	2943.238	1857.096
6	8000000	3925000	2875000	2473.939	1400	9882181	4498234	3813680	2835.251	1857.096
7	8000000	3790654	3009346	2327.040	1400	9882181	4320025	3991889	2652.015	1857.096
8	8000000	3975439	2824561	2531.832	1400	9882181	4565141	3746773	2907.395	1857.096
9	8000000	4024138	2775862	2589.269	1400	9882181	4629740	3682173	2978.934	1857.096
10	8000000	4000000	2800000	2560.606	1400	9882181	4597721	3714192	2943.238	1857.096
11	8000000	4024138	2775862	2589.269	1400	9882181	4629740	3682173	2978.934	1857.096
12	8000000	3987692	2775862	2565.818	1400	9882181	4629740	3682173	2978.934	1857.096
13	8000000	4000000	2800000	2560.606	1400	9882181	4597721	3714192	2943.238	1857.096
14	8000000	3975439	2824561	2531.832	1400	9882181	4565141	3746773	2907.395	1857.096
15	8000000	4024138	2775862	2589.269	1400	9882181	4629740	3682173	2978.934	1857.096
16	8000000	4024138	2775862	2589.269	1400	9882181	4629740	3682173	2978.934	1857.096
17	8000000	4047863	2752137	2617.823	1400	9882181	4661212	3650702	3014.486	1857.096
18	8000000	4047863	2752137	2617.823	1400	9882181	4661212	3650702	3014.486	1857.096
NPV	**307721700**					**393397600**				

As it could be seen in the results of the deterministic and stochastic models shown in Table 6, the three stages of mine, processing, and refinery except for the first metal use their maximum capacities during the mine's life. There is some reason why the refinery unit did not work with its maximum capacity in producing the first metal. Based upon the assumed data in Table 3, the profit (price minus cost) of the first metal is approximately five times lower than the profit of the second one. For this reason and based on the objective function as the NPV maximization, first, the processing unit capacity was assigned to supply the material required for producing the maximum possible amount of the second metal. Then the rest of the processing capacity was assigned to produce the first concentrate as a feed to be sent for the refinery unit to produce the first metal.

It can also be resulted that the rate of the materials sent from the mine to the processing plant during the mine’s life is the same. The annual rate of the sum of the first and second products sent from the processing unit to the refinery units during the mine's life is the same, as well. In addition, the rate of the second metal produced at the refinery during the mine’s life is the same; whereas, no specific trend can be seen in the results of the first metal because these rates are associated with frequent highs and lows.

Clearly, the results show that the rate of materials sent annually from the mine to the processing unit and also from the processing to the refinery units during different years is higher in the stochastic model than in the deterministic model. Furthermore, the rate of metal produced annually at the stochastic model is by far more than the same value in the deterministic model. For this reason, NPV in the stochastic model is more than the deterministic model, and implies that the provided stochastic model achieves more favorable results than the deterministic model. In this case, in order to reduce the costs caused by over-production advances towards a relative increase in the rate of sales, and as a result of the increase in revenue from this process, this model leads to an increase in NPV.

The curve of changes in the production of each metal is provided per the various parameters. Figures 2 and 3 show the trend of changes in production of the first and second pure metals in various years in the deterministic and stochastic models. According to Figure 2, both the deterministic and stochastic curves show an uptrend in the production rates of the first metals over time, in line with optimization of NPV and covering extant costs. Interestingly, in the production of the first metal based upon both the deterministic and stochastic models, the maximum and minimum production rates were seen in the same years. It is clear that in the deterministic model, the maximum value of the first metal has occurred in years 17 and 18 with a value of 2617.82 tons, and the minimum value of 2267.34 tons was achieved in year 3. For this metal in the stochastic model, the maximum value of production in years 17 and 18 occurred with a value of 3014.49 tons, and the minimum with a value of 2577.47 tons occurred in year 3. As illustrated in Figure 3, both the deterministic and stochastic curves for the second metal are constant. In this case, the refinery unit produced the second metal by its maximum capacity.

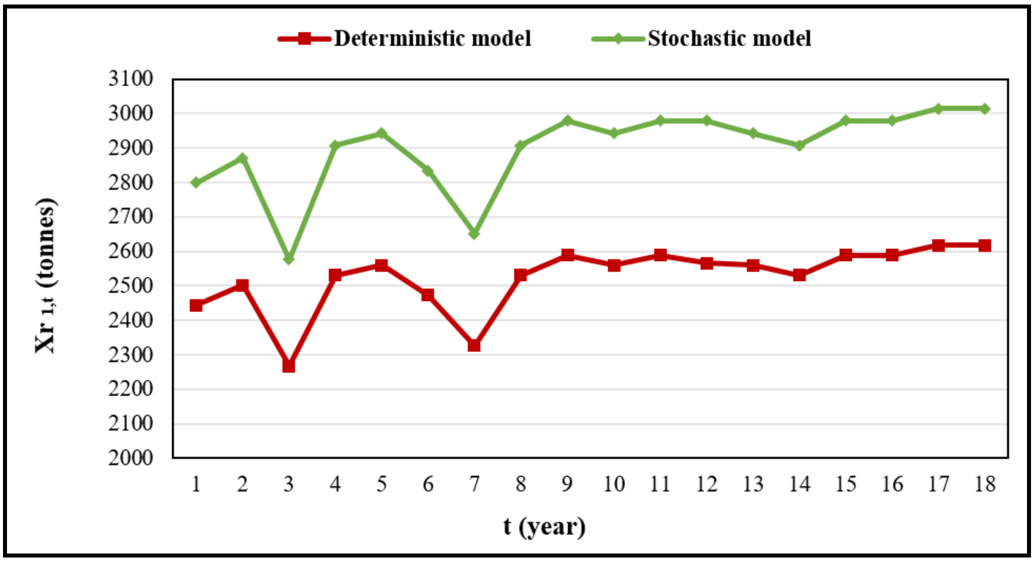

Figure 2. Change curve of first metal production per various years in deterministic and stochastic models.

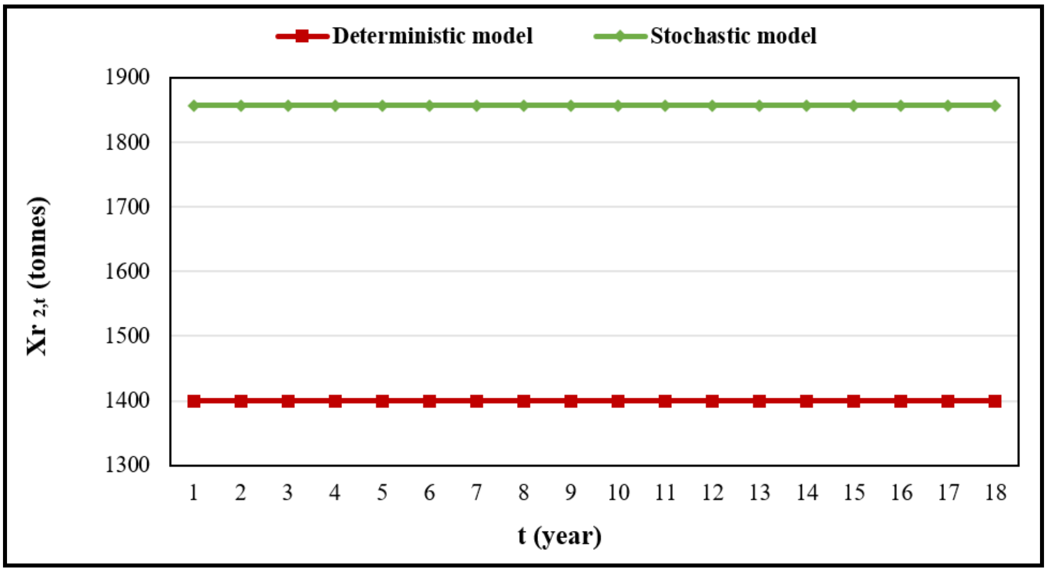

Figure 3. Change curve of second metal production per various years in deterministic and stochastic models.

The production rates determined in completing the investigation of the process of producing the first and second metals compared to the capacity of the refinery stage are shown in Figures 4 and 5. According to these figures, the stochastic model attained much better results than the deterministic model. Figure 4 shows that the values obtained for the first metal in both the deterministic and stochastic models were obviously less than the refinery capacity, whereas, as shown in Figure 5, the resulting values for the second metal in both models were equal to the maximum refinery capacity. Figure 4 shows that by increasing the capacity of the refinery stage and due to the randomness of this parameter and its ability to increase in the stochastic chance-constrained programming, the production rate is higher in the stochastic model than in the deterministic model.

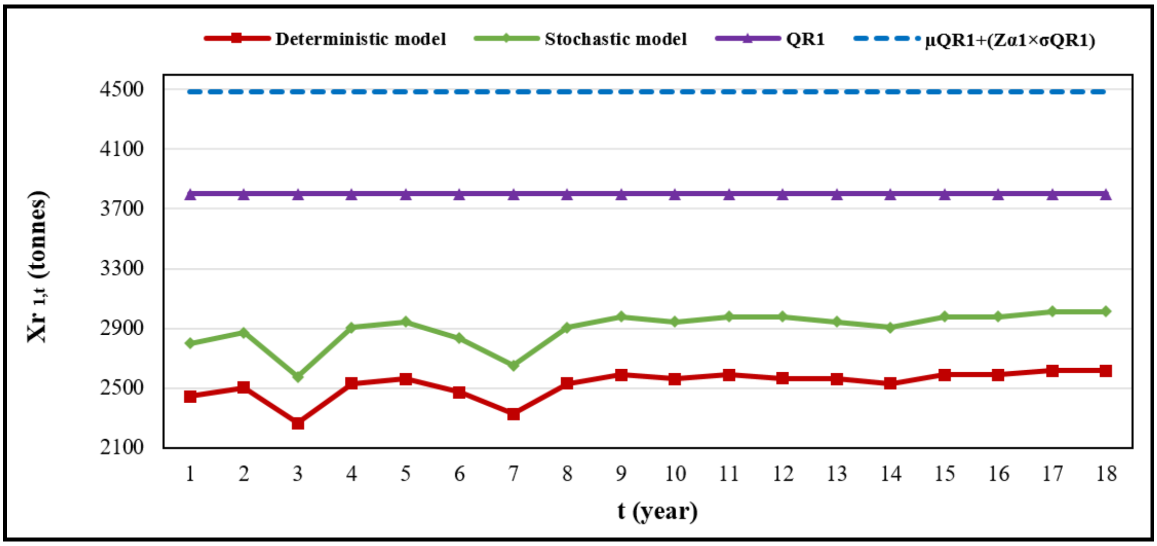

Figure 4. Production rate of first metal and refinery capacity in deterministic and stochastic models.

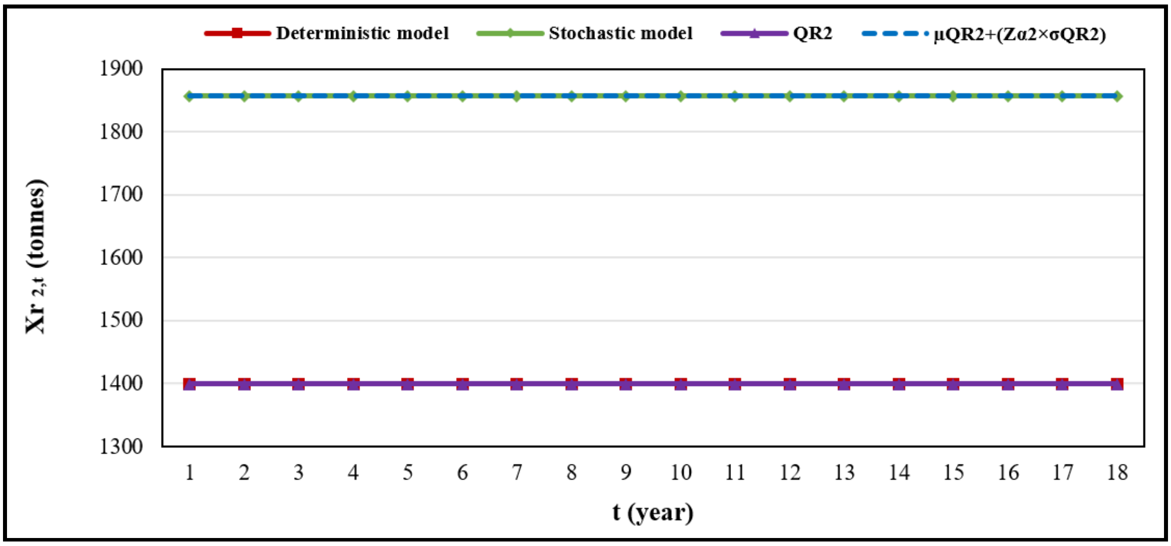

Figure 5. Production rate of second metal and refinery capacity in deterministic and stochastic models.

4. Conclusions

Since the parameters such as grade, price, and capacity of the various parts of a metal production process in bimetallic deposits are not fixed in real and operational conditions, a stochastic model was developed under uncertain conditions for these parameters. First, a deterministic model was presented to determine the mine production strategy. Then given the uncertainty of the studied parameters, the stochastic model of optimizing the production strategy was presented. In the case example, the two models mentioned above for an assumed solved deposit were far better than the deterministic model, and NPV resulting from this model in the format of planning strategy for the production of a bimetallic deposit was far better than that of the deterministic models. The results obtained also showed that by sending more materials from the mine to the processing plant, based on the stochastic programming model, the rate of metal produced increased. It is indicative of the fact that by considering the uncertainties, the final metal produced at the refinery stage is increased, and its values during different years are significantly higher than the same values in the deterministic model.

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راهبرد بهینه تولید کانسارهای دوفلزی تحت عدم‌قطعیت‌های فنی و اقتصادی با استفاده از برنامه‌ریزی تصادفی محدودیت شانس

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چکیده:

برای سازگاری با واقعیت، تمامی تصمیم‌های کلان درباره برنامه‌ریزی تولید بلندمدت معدن باید به طور همزمان و تحت شرایط عدم قطعیت پارامترهای تعیین‌کننده اتخاذ شود. در این پژوهش، با بهره‌گیری از مزایای برنامه‌ریزی محدودیت شانس، مدلی تصادفی برای تعیین راهبرد بهینه تولید معادن روباز دوفلزی در شرایط قطعیت و عدم قطعیت ارائه شد. عدم قطعیت در پارامترهای عیار، قیمت به ازای هر محصول و ظرفیت‌های مراحل مختلف فرآیند تولید محصول نهایی در نظر گرفته شد. نتایج حاصل از حل مدل‌های قطعی و تصادفی ارائه‌شده نشان داد که سازگاری و کارکرد مدل تصادفی بیش‌تر از سایر مدل‌ها بوده است.

کلمات کلیدی: کانسارهای دوفلزی، عدم قطعیت، عیار، قیمت، ظرفیت.